Ex 10.1

Answer 1. $5x^2 - 9x + 4$ is divided by (x-2) Putting x-2=0, we get: x=2 Substituting this value of x in the equation, we get 5x2x2 - 9x2 + 4 = 20 - 18 + 4=6 $5x^3 - 7x^2 + 3$ is divided by (x-1) Putting x-1=0, we get: x=1 Substituting this value of x in the equation, we get 5x1x1x1 - 7x1x1 + 3= 5 - 7 + 3=1 $8x^2 - 2x + 1$ is divided by (2x+1)Putting 2x+1=0, we get: $x=-\frac{1}{2}$ Substituting this value of x in the equation, we get $8x\left(-\frac{1}{2}\right)x\left(-\frac{1}{2}\right)-2x\left(-\frac{1}{2}\right)+1$ = 2+1+1 - 4 $x^{3} + 8x^{2} + 7x - 11$ is divisible by (x+4) Putting x+4=0, we get: x=-4Substituting this value of x in the equation, we get (-4)x(-4)x(-4) + 8x(-4)x(-4) + 7x(-4) - 11=-64+128-28-11 =25 $2x^3 - 3x^2 + 6x - 4$ is divisible by (2x-3) Putting 2x-3=0, we get: $x=\frac{3}{2}$ Substituting this value of x in the equation, we get $2x\frac{3}{5}x\frac{3}{5}x\frac{3}{5}-3x\frac{3}{5}x\frac{3}{5}+6x\frac{3}{5}-4$ $=\frac{27}{4}-\frac{27}{4}+9-4$ - 5



Answer 2.
(x-2) is a factor of
$$2x^3 - 7x - 2$$

x-2=0 \Rightarrow x=2
Substituting this value, we get
f(2) = $2x2x2x2 - 7x2 - 2 = 0$
Hence (x-2) is a factor of $2x^3 - 7x - 2$
(2x+1) is a factor of $4x^3 + 12x^2 + 7x + 1$
 $2x+1=0 \Rightarrow x= -\frac{1}{2}$
Substituting this value, we get
f $\left(-\frac{1}{2}\right) = 4x\left(-\frac{1}{2}\right)x\left(-\frac{1}{2}\right)x\left(-\frac{1}{2}\right) + 12x\left(-\frac{1}{2}\right)x\left(-\frac{1}{2}\right) + 7x\left(-\frac{1}{2}\right) + 1$
 $= 0$
Hence (2x+1) is a factor of $4x^3 + 12x^2 + 7x + 1$
(3x-2) is a factor of $18x^3 - 3x^2 + 6x - 8$
 $3x-2=0 \Rightarrow x=\frac{2}{3}$
Substituting this value, we get
f $\left(\frac{2}{3}\right) = 18x\left(\frac{2}{3}\right)x\left(\frac{2}{3}\right)x\left(\frac{2}{3}\right) - 3x\left(\frac{2}{3}\right)x\left(\frac{2}{3}\right) + 6x\left(\frac{2}{3}\right) - 8$
 $= \frac{16}{3} - \frac{4}{3} + 4 - 8$
 $= 4 + 4 - 8$
 $= 0$
Hence (3x-2) is a factor of $18x^3 - 3x^2 + 6x - 8$.
(2x-1) is a factor of $6x^3 - x^2 - 5x + 2$
 $2x-1=0 \Rightarrow x=\frac{1}{2}$
Substituting this value, we get
f $\left(\frac{1}{2}\right) = 6x\frac{1}{2}x\frac{1}{2}x\frac{1}{2} - \frac{1}{2}x\frac{1}{2} - 5x\frac{1}{2} + 2$
 $= \frac{3}{4} - \frac{1}{4} - \frac{5}{2} + 2$
 $= \frac{1}{2} - \frac{5}{2} + 2 - -2 + 2 = 0$
Hence (2x-1) is a factor of $6x^3 - x^2 - 5x + 2$
(x-3) is a factor of $5x^2 - 21x + 18$.

Answer 3. $\Rightarrow x = -2 \dots (i)$ $(2x-1) \Rightarrow x = \frac{1}{2} \dots (ii)$ Putting (i) in polynomial, we get f(-2) = 2x(-2)x(-2)x(-2) + ax(-2)x(-2) + bx(-2) + 10 = 0 $\Rightarrow -16 + 4a - 2b + 10 = 0$ $\Rightarrow a = \frac{b}{2} + \frac{3}{2} \dots (iii)$

Putting (ii) in polynomial , we get

$$\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + a \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + b \times \left(\frac{1}{2}\right) + 10 = 0 \Rightarrow \frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 10 = 0 \Rightarrow a = -2b - 41.....(iv) \text{ combining (iii) and (iv), we get, } \frac{b}{2} + \frac{3}{2} = a = -2b - 41 \Rightarrow \frac{b+3}{2} = -2b - 41 \Rightarrow b+3 = -4b - 82 \Rightarrow 5b = -85 \Rightarrow b = -17 ind a = -7 \Rightarrow a = -7, b = -17$$

Answer 4.

x-1=0 ⇒ x=1 and remainder is 2m Substituting this value, we get : $f(1) = 1 \times 1 \times 1 + 5 \times 1 \times 1 - m \times 1 + 6 = 2m$ ⇒3m=12 ⇒ m=4

Answer 5.

x-2=0 ⇒ x=2 and remainder is 0 Substituting this value, we get : f(2) = 2x2x2 + 3x2x2 - mx2 + 4 = 0 ⇒2m=24 ⇒ m=12

Answer 6.

 $(x-1) \Rightarrow x=1 \dots (i)$ $(x-2) \Rightarrow x = 2 \dots (ii)$ Putting (i) in polynomial, we get $f(1) = 1 \times 1 \times 1 - p \times 1 \times 1 + 14 \times 1 - q = 0$ $\Rightarrow p + q = 15$ $\Rightarrow p=15 - q \dots (iii)$ Putting (ii) in polynomial, we get $f(2) = 2 \times 2 \times 2 - p \times 2 \times 2 + 14 \times 2 - q = 0$ $4p + q = 36, \Rightarrow q = 36 - 4p \dots (iv)$ Combining (iii) and (iv), we get, p = 15 - (36 - 4p) $\Rightarrow p = 15 - 36 + 4p$ $\Rightarrow 3p = 21$ $q = 36 - 4 \times 7 = 8$



Answer 7.
3)
$$\Rightarrow x = -\frac{3}{2}....(i)$$

(x+2) $\Rightarrow x = -2....(ii)$
Putting (i) in polynomial, we get
 $f\left(-\frac{3}{2}\right) = ax\left(-\frac{3}{2}\right)x\left(-\frac{3}{2}\right)x\left(-\frac{3}{2}\right) + 3x\left(-\frac{3}{2}\right)x\left(-\frac{3}{2}\right) + bx\left(-\frac{3}{2}\right) - 3$
 $= 0$
 $-27a + 54 - 12b - 24 = 0$
 $\Rightarrow 27a = -12b + 30....(iii)$
Putting (ii) in polynomial, and remainder is -3 we get
 $f(-2) = ax(-2)x(-2)x(-2) + 3x(-2)x(-2) + bx(-2) - 3 = -3$
 $b = 6 - 4a(iv)$
Combining (iii) and (iv), we get,
 $27a = -12x (6-4a) + 30$
 $\Rightarrow 27a = -72 + 48a + 30$,
 $\Rightarrow a = 2, b = 6 - 4x^2 = -2$
 $a = 2, b = -2$





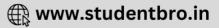
Answer 8.

 $\begin{array}{l} (x+2) \Rightarrow x=-2 \dots (i) \\ (x+1) \Rightarrow x=-1 \dots (ii) \\ \\ \text{Putting (i) in polynomial , we get} \\ f(-2) = (-2)x(-2)x(-2) - 2x(-2)x(-2) + mx(-2) + n = 0 \\ \\ \Rightarrow -8 - 2m + n = 0 \\ \\ \Rightarrow n=2m + 16 \dots (iii) \\ \\ \\ \text{Putting (ii) in polynomial, and remainder is 9 we get} \end{array}$

$$\begin{array}{l} f(-1) = (-1) \times (-1) \times (-1) - 2 \times (-1) \times (-1) + m \times (-1) + n = 9 \\ \Rightarrow -1 - 2 - m + n = 9, \\ \Rightarrow m = n - 12 \dots (iv) \\ \\ \text{Combining (iii) and (iv), we get,} \\ n = 2 \times (n - 12) + 16, \\ \Rightarrow n = 8. \\ \\ \text{Hence, } m = n - 12 = 8 - 12 = -4 \end{array}$$

m = -4, n= 8



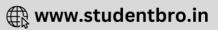


Answer 9.

$$(2x+1) \Rightarrow x = -\frac{1}{2}$$

Solving Equation (i), we get
$$f\left(-\frac{1}{2}\right) = 2x\left(-\frac{1}{2}\right)x\left(-\frac{1}{2}\right) - 5x\left(-\frac{1}{2}\right) + a = 0$$
$$\Rightarrow \frac{1}{2} + \frac{5}{2} + a = 0$$
$$\Rightarrow a = -3$$
$$g\left(-\frac{1}{2}\right) = 2x\left(-\frac{1}{2}\right)x\left(-\frac{1}{2}\right) + 5x\left(-\frac{1}{2}\right) + b = 0$$
$$\Rightarrow \frac{1}{2} - \frac{5}{2} + b = 0$$
$$\Rightarrow b = 2$$
$$\Rightarrow a = -3, b = 2$$





Answer 10.
x=-3....(i)
:-1)
$$\Rightarrow x = \frac{1}{2}$$
....(ii)
ting (i) in polynomial, we get
) = ax(-3)x(-3)x(-3) + bx(-3)x(-3) + (-3) - a = 0
:7a + 9b - 3 - a = 0

ng (ii) in polynomial, we get

$$\int = ax \left(\frac{1}{2}\right) x \left(\frac{1}{2}\right) x \left(\frac{1}{2}\right) + bx \left(\frac{1}{2}\right) x \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) - a = 0$$

$$+ \frac{b}{4} + \frac{1}{2} - a = 0$$

$$= \frac{7a}{2} - 2....(iv)$$

ining (iii) and (iv), we get,

$$\frac{7a}{2} - 2 - \frac{3}{28}$$

= 63a - 42
6
 $\frac{7 \times 6}{2} - 2 = 21 - 2 = 19$

i, b=19

these values in polynomial, we get

 $6x^3 + 19x^2 + x - 6$

equation becomes (x+3)(2x-1)(3x+2) = 0



Answer 11.

 $(x-2) = 0 \Rightarrow x=2$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be subtracted so that polynomial is exactly subtracted by the factor.

f(2) = 2x2 + 2 + 1 - a=0 ⇒a = 7 Henœ answer =7

Answer 12.

 $(x-1)=0 \Rightarrow x=1$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be added so that polynomial is exactly subtracted by the factor.

 $f(1) = 2 \times 1 \times 1 \times 1 - 3 \times 1 \times 1 + 7 \times 1 - 8 + a = 0$

⇒a = 2

Hence answer =2

Answer 13.

$$(2\times-5)=0 \Rightarrow \times=\frac{5}{2}$$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be subtracted so that polynomial is exactly subtracted by the factor.

$$f\left(\frac{5}{2}\right) = 2 \times \left(\frac{5}{2}\right) \times \left(\frac{5}{2}\right) \times \left(\frac{5}{2}\right) - 5 \times \left(\frac{5}{2}\right) \times \left(\frac{5}{2}\right) + 8 \times \left(\frac{5}{2}\right) - 17 - a = 0$$

$$\Rightarrow \frac{125}{4} - \frac{125}{4} + 20 - 17 - a = 0$$

$$\Rightarrow a = 3$$

$$- + 20 - 17 - a = 0$$
Hence answer = 3

Answer 14.

$$(2x-1) \Rightarrow x = \frac{1}{2}$$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be added so that polynomial is exactly subtracted by the factor.

$$f\left(\frac{1}{2}\right) = 12 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + 16 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) - 5 \times \left(\frac{1}{2}\right) - 8 + a = 0$$

$$\Rightarrow \frac{3}{2} + 4 - \frac{5}{2} - 8 + a = 0$$

$$\Rightarrow a = 5$$

+ 4 - - 8 + a =0Hence answer =5

Answer 15.

We know that $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \dots$ (i)

And if we put $a-b=0 \Rightarrow a=b$, and substitute this to the polynomial, we get:

$$f(x) = 0 + (a-c)^3 + (c-a)^3 = (a-c)^3 - (a-c)^3 = 0$$

Hence, (a-b) is a factor. \Rightarrow a=b (ii)

Substituting (i) in problem polynomial, we get

$$f(x) = 0 + (b^3 - 3b^2c + 3bc^2 - c^3) + (c^3 - 3c^2a + 3ca^2 - a^3)$$

$$= -3b^2c + 3bc^2 - 3ca^2 + 3ca^2$$

If we put $b-c=0 \Rightarrow b=c$, and substitute this to the polynomial, we get:

$$f(b=c), \Im(-c^2xc + cxc^2 - cxc^2 + cxc^2) = 0$$

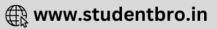
Hence, till now factors are 3x(a-b)x(b-c) (iii)

Similarly if we had put c=a, we would have got similar result.

From (ii), (iii), and (iv), we get

3(a-b)(b-c)(c-a) is a complete factorization of the given polynomial.





Answer 16.

If p-q is assumed to be factor, then p=q. Substituting this in problem polynomial, we get:

$$f(p=q) = (p-r)^3 + (r-p)^3$$
$$= (p-r)^3 + (-(p-r))^3$$
$$= (p-r)^3 - (p-r)^3$$
$$= 0$$

Hence, (p-q) is a factor.

Answer 17.

If x-y is assumed to be factor, then x=y. Substituting this in problem polynomial, we get:

$$f(x=y) = yz(y^2-z^2) + zy(z^2-y^2) + yy(y^2-y^2)$$
$$= yz(y^2-z^2) + zy(-(y^2-z^2)) + 0$$
$$= yz(y^2-z^2) - yz(y^2-z^2) = 0$$

Hence, (x-y) is a factor.

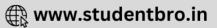
Answer 18.

If x-3 is assumed to be factor, then x=3. Substituting this in problem polynomial, we get:

f(3) = 3x3x3 - 3x3 - 9x3 + 9 = 0

Hence its proved that x-3 is a factor of the polynomial.





Answer 19.

If x + 1 is assumed to be factor, then x = -1. Substituting this in problem polynomial, we get:

f(-1) = (-1)x(-1)x(-1) - 6x(-1)x(-1) + 5x(-1) + 12 = 0

Hence (x+1) is a factor of the polynomial.

Multiplying (x+1) by x^2 , we get $x^3 + x^2$, hence we are left with $-7x^2 + 5x + 12$ (and 1^{st} part of factor as x^2).

Multiplying (x+1) by -7x, we get $-7x^2 - 7x$, hence we are left with 12x + 12 (and 2^{nd} part of factor as -7x).

Multiplying (x+1) by 12, we get 12x + 12, hence we are left with 0 (and 3^{rd} part of factor as 12).

Hence complete factor is $(x+1)(x^2-7x+12)$.

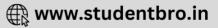
Further factorizing $(x^2-7x+12)$, we get:

 $x^{2} - 3x - 4x + 12 = 0$

⇔ (x-4)(x-3)=0

Hence answer is (x+1)(x-4)(x-3) = 0





Answer 20.

If 5x - 4 is assumed to be factor, then $x = \frac{4}{5}$. Substituting this in problem polynomial, we get:

$$f\left(\frac{4}{5}\right) = 5 \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) - 4 \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) - 5 \times \left(\frac{4}{5}\right) + 4$$
$$= \frac{64}{25} - \frac{64}{25} - 4 + 4$$
$$= 0$$

Hence (5x-4) is a factor of the polynomial.

Multiplying (5x-4) by x^2 , we get $5x^3 - 4x^2$, hence we are left with -5x + 4 (and 1^{st} part of factor as x^2).

Multiplying (5x-4) by -1, we get -5x + 4, hence we are left with O(and 2^{nd} part of factor as -7x).

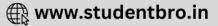
Hence complete factor is $(5x-4)(x^2-1)$.

Further factorizing (\times^2-1) , we get:

=>(x-1)(x+1)=0

Hence answer is (5x-4)(x-1)(x+1) = 0





Answer 21.

Given f(x) = (x-1)(x-2)+(-2x+5) $=(x^2-3x+2)+(-2x+5)$ $f(x)=x^2-5x+7$ Substituting x=1 f(x) = 1-5+7 = 3when f(x) is divided by (x-1), remainder = 3 substituting x=2 f(x) = 4-10+7 = 1when f(x) is divided by (x-2), remainder = 1 $\frac{x^2-5x+7}{x^2-3x+2} = 1\frac{(-2x+5)}{(x-1)(x-2)}$ and when f(x) is divided by (x-1)(x-2), remainder = (-2x+5).

Answer 22. (hen x + 1 is a factor, we can substitute x=-1 to evaluate values. ...(i) When x - 2 is a factor, we can substitute x=2 to evaluate values. ...(ii) When 2x - 1 is a factor, we can substitute $x=\frac{1}{2}$ to evaluate values. ...(iii)

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ubstituting (i), we get

-1) =
$$ax(-1)^4$$
 + (-1)³ + b(-1)² - 4(-1) + c = 0
⇒ a + b + c = -3,
⇒ a = -b -c - 3....(iv)

ubstituting (ii), we get

$$\Rightarrow f(2) = ax(2)^4 + (2)^3 + b(2)^2 - 4(2) + c = 0$$

$$\Rightarrow$$
 16a + 4b + c = 0(v)

ubstituting (iil), we get

$$\Rightarrow f(\frac{1}{2}) = ax(\frac{1}{2})^{4} + (\frac{1}{2})^{3} + b(\frac{1}{2})^{2} - 4(\frac{1}{2}) + c = 0$$

$$\Rightarrow \frac{a}{16} + \frac{b}{4} + c = 2 - \frac{1}{8}$$

$$\Rightarrow a + 4b + 16c = 30....(vi)$$

Utting (iv) in (v) and (vi), we get:

$$5a + 4b + c = 0; \Rightarrow 16x(-b - c - 3) + 4b + c = 0$$

$$\Rightarrow -12b - 15c - 48 = 0 \Rightarrow 4b + 5c = -16$$

$$\Rightarrow b = -4 - \frac{5c}{4}....(vii)$$

$$+ 4b + 16c = 30 \Rightarrow (-b - c - 3) + 4b + 16c = 30$$

$$\Rightarrow 3b + 15c = 33....(viii)$$

Putting (vii) in (viii), we get,

$$\Rightarrow 3x(-4 - \frac{5c}{4}) + 15c = 33,$$

$$\Rightarrow Solving this, we get$$

⇒c=4

Putting this value of c in (viii), we get:



