Chapter 10. Remainder And Factor Theorems

Ex 10.1

Answer 1.

$5x^2 - 9x + 4$ is divided by (x-2)
Putting x-2=0, we get: x=2
Substituting this value of x in the equation, we get
$5 \times 2 \times 2 - 9 \times 2 + 4 = 20 - 18 + 4$
$= 6$

$5x^3 - 7x^2 + 3$ is divided by (x-1)
Putting x-1=0, we get: x=1
Substituting this value of x in the equation, we get
$5 \times 1 \times 1 \times 1 - 7 \times 1 \times 1 + 3$
$= 5 - 7 + 3$
$= 1$

$8x^2 - 2x + 1$ is divided by (2x+1)
Putting 2x+1=0, we get: $x = -\frac{1}{2}$
Substituting this value of x in the equation, we get
$$8 \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) - 2 \times \left(-\frac{1}{2}\right) + 1$$
$= 2 + 1 + 1$
$= 4$

$x^3 + 8x^2 + 7x - 11$ is divisible by (x+4)
Putting x+4=0, we get: x=-4
Substituting this value of x in the equation, we get
$(-4) \times (-4) \times (-4) + 8 \times (-4) \times (-4) + 7 \times (-4) - 11$
$= -64 + 128 - 28 - 11$
$= 25$

$2x^3 - 3x^2 + 6x - 4$ is divisible by (2x-3)
Putting 2x-3=0, we get: $x = \frac{3}{2}$
Substituting this value of x in the equation, we get
$$2 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} - 3 \times \frac{3}{2} \times \frac{3}{2} + 6 \times \frac{3}{2} - 4$$
$= \frac{27}{4} - \frac{27}{4} + 9 - 4$
$= 5$

Answer 2.

(x-2) is a factor of $2x^3 - 7x - 2$

$x-2=0 \Rightarrow x=2$

Substituting this value, we get

f(2) = 2x2x2x2 – 7x2 -2 = 0

Hence (x-2) is a factor of 2x3– 7x -2

(2x+1) is a factor of $4x^3 + 12x^2 + 7x + 1$

$2x+1=0 \Rightarrow x=-\frac{1}{2}$

Substituting this value, we get

$$f\left(-\frac{1}{2}\right) = 4\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)+12\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)+7\times\left(-\frac{1}{2}\right)+1$$

$$= 0$$

Hence (2x+1) is a factor of $4x^3 + 12x^2 + 7x + 1$

) (3x-2) is a factor of $18x^3 - 3x^2 + 6x - 8$

$3x-2=0 \Rightarrow x=\frac{2}{3}$

Substituting this value, we get

$$f\left(\frac{2}{3}\right) = 18\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)-3\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)+6\times\left(\frac{2}{3}\right)-8$$

$$= \frac{16}{3}-\frac{4}{3}+4-8$$

$$= 4+4-8$$

$$= 0$$

Hence (3x-2) is a factor of $18x^3 - 3x^2 + 6x - 8$.

) (2x-1) is a factor of $6x^3 - x^2 - 5x + 2$

$2x-1=0 \Rightarrow x=\frac{1}{2}$

Substituting this value, we get

$$f\left(\frac{1}{2}\right) = 6\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}-\frac{1}{2}\times\frac{1}{2}-5\times\frac{1}{2}+2$$

$$= \frac{3}{4}-\frac{1}{4}-\frac{5}{2}+2$$

$$= \frac{1}{2}-\frac{5}{2}+2 = -2+2 = 0$$

Hence (2x-1) is a factor of $6x^3 - x^2 - 5x + 2$

(x-3) is a factor of $5x^2 - 21x + 18$.

Answer 3.

$\Rightarrow x=-2 \; .... \text{(i)}$

$(2x-1) \Rightarrow x = \frac{1}{2} \; .... \text{(ii)}$

Putting (i) in polynomial , we get

$f(-2) = 2\times(-2)\times(-2)\times(-2) + a\times(-2)\times(-2) + b\times(-2) + 10 = 0$

$\Rightarrow -16 + 4a - 2b + 10 = 0$

$\Rightarrow a = \frac{b}{2} + \frac{3}{2} \quad ....\text{(iii)}$

Putting (ii) in polynomial , we get

$\left(\frac{1}{2}\right) = 2\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) + a\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) + b\times\left(\frac{1}{2}\right) + 10 = 0$

$\Rightarrow \frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 10 = 0$

$\Rightarrow a = -2b - 41 ....\text{(iv)}$

Combining (iii) and (iv), we get,

$\frac{b}{2} + \frac{3}{2} = a = -2b - 41$

$\Rightarrow \frac{b+3}{2} = -2b - 41$

$\Rightarrow b + 3 = -4b - 82$

$\Rightarrow 5b = -85$

$\Rightarrow b = -17$

and $a = -7$

$\Rightarrow a = -7 , b = -17$

Answer 4.

$x-1=0 \Rightarrow x=1$ and remainder is $2m$

Substituting this value, we get :

$f(1) = 1\times1\times1 + 5\times1\times1 - m\times1 + 6 = 2m$

$\Rightarrow 3m=12$

$\Rightarrow m=4$

Answer 5.

$x-2=0 \Rightarrow x=2$ and remainder is 0

Substituting this value, we get :

$f(2) = 2x2x2 + 3x2x2 - mx2 + 4 = 0$

$\Rightarrow 2m=24$

$\Rightarrow m=12$

Answer 6.

$(x-1) \Rightarrow x=1$ .... (i)

$(x-2) \Rightarrow x = 2$ .... (ii)

Putting (i) in polynomial , we get

$f(1) = 1x1x1 - px1x1 + 14x1 - q = 0$

$\Rightarrow p + q = 15$

$\Rightarrow p=15 - q$....(iii)

Putting (ii) in polynomial , we get

$f(2) = 2x2x2 - px2x2 + 14x2 - q = 0$

$4p + q = 36, \Rightarrow q = 36 - 4p$ ....(iv)

Combining (iii) and (iv), we get,

$p = 15 - (36-4p)$

$\Rightarrow p = 15 - 36 + 4p$

$\Rightarrow 3p=21$

$q = 36-4x7 = 8$

$\Rightarrow p = 7, q=8$

Answer 7.

3) ⇒ $x = -\frac{3}{2}$ .... (i)

(x+2) ⇒ x = -2 .... (ii)

Putting (i) in polynomial , we get

$$f\left(-\frac{3}{2}\right) = a\times\left(-\frac{3}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{3}{2}\right)+3\times\left(-\frac{3}{2}\right)\times\left(-\frac{3}{2}\right)+b\times\left(-\frac{3}{2}\right)-3$$

$$= 0$$

$-27a + 54 - 12b - 24 = 0$

⇒ $27a = -12b + 30$....(iii)

Putting (ii) in polynomial, and remainder is -3 we get

$f(-2) = a\times(-2)\times(-2)\times(-2) + 3\times(-2)\times(-2) + b\times(-2) - 3 = -3$

$b = 6 - 4a$ .....(iv)

Combining (iii) and (iv), we get,

$27a = -12\times(6-4a) + 30$

⇒ $27a = -72 + 48a + 30$,

⇒ $a = 2$, $b = 6-4\times2 = -2$

$a = 2$, $b = -2$

Answer 8.

$(x+2) \Rightarrow x = -2$ .... (i)

$(x+1) \Rightarrow x = -1$ .... (ii)

Putting (i) in polynomial , we get

$f(-2) = (-2)x(-2)x(-2) - 2x(-2)x(-2) + mx(-2) + n = 0$

$\Rightarrow -8 - 8 - 2m + n = 0$

$\Rightarrow n = 2m + 16$....(iii)

Putting (ii) in polynomial, and remainder is 9 we get

$f(-1) = (-1)x(-1)x(-1) - 2x(-1)x(-1) + mx(-1) + n = 9$

$\Rightarrow -1 - 2 - m + n = 9,$

$\Rightarrow m = n - 12$ .....(iv)

Combining (iii) and (iv), we get,

$n = 2x(n - 12) + 16,$

$\Rightarrow n = 8.$

Hence, $m = n - 12 = 8 - 12 = -4$

$m = -4, n = 8$

Answer 9.

$(2x+1) \Rightarrow x=-\frac{1}{2}$

Solving Equation (i), we get

$$f\left(-\frac{1}{2}\right)=2\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)-5\times\left(-\frac{1}{2}\right)+a=0$$
$$\Rightarrow \frac{1}{2}+\frac{5}{2}+a=0$$
$$\Rightarrow a=-3$$

$$g\left(-\frac{1}{2}\right)=2\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)+5\times\left(-\frac{1}{2}\right)+b=0$$
$$\Rightarrow \frac{1}{2}-\frac{5}{2}+b=0$$
$$\Rightarrow b=2$$

$$\Rightarrow a=-3,\ b=2$$

Answer 10.

x=-3 .... (i)

:-1) $\Rightarrow x = \frac{1}{2}$....(ii)

:ting (i) in polynomial , we get

) = ax(-3)x(-3)x(-3) + bx(-3)x(-3) + (-3) - a = 0

:7a + 9b - 3 - a = 0

$= \frac{9b}{28} - \frac{3}{28}$...(iii)

ng (ii) in polynomial , we get

$) = a \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + b \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) - a = 0$

$+ \frac{b}{4} + \frac{1}{2} - a = 0$

$= \frac{7a}{2} - 2$....(iv)

ining (iii) and (iv), we get,

$\times \left(\frac{7a}{2} - 2\right) - \frac{3}{28}$

$= 63a - 42$

6

$\frac{7 \times 6}{2} - 2 = 21 - 2 = 19$

, b=19

these values in polynomial, we get

$6x^3 + 19x^2 + x - 6$

equation becomes (x+3)(2x-1)(3x+2) = 0

Answer 11.

(x-2) =0$\Rightarrow$ x=2

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be subtracted so that polynomial is exactly subtracted by the factor.

f(2) = 2x2 + 2 + 1 - a=0

$\Rightarrow$a = 7

Hence answer =7

Answer 12.

(x-1)=0 $\Rightarrow$ x=1

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be added so that polynomial is exactly subtracted by the factor.

f(1) = 2x1x1x1 – 3x1x1 + 7x1 - 8 + a=0

$\Rightarrow$a = 2

Hence answer =2

Answer 13.

$(2x-5)=0 \Rightarrow x=\frac{5}{2}$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be subtracted so that polynomial is exactly subtracted by the factor.

$$f\left(\frac{5}{2}\right)=2\times\left(\frac{5}{2}\right)\times\left(\frac{5}{2}\right)\times\left(\frac{5}{2}\right)-5\times\left(\frac{5}{2}\right)\times\left(\frac{5}{2}\right)+8\times\left(\frac{5}{2}\right)-17-a=0$$
$$\Rightarrow \frac{125}{4}-\frac{125}{4}+20-17-a=0$$
$$\Rightarrow a=3$$

\- + 20 - 17 – a =0Hence answer =3

Answer 14.

$(2x-1) \Rightarrow x= \frac{1}{2}$

When we substitute this value in the polynomial, whatever we get as a remainder (say a) should be added so that polynomial is exactly subtracted by the factor.

$$f\left(\frac{1}{2}\right) = 12\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)+16\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)-5\times\left(\frac{1}{2}\right)-8+a=0$$

$$\Rightarrow \frac{3}{2}+4-\frac{5}{2}-8+a=0$$

$$\Rightarrow a = 5$$

+ 4 - - 8 + a =0Hence answer =5

Answer 15.

We know that $(a - b)^3 = a^3 - 3a^2b + 3\,ab^2 - b^3$ ..... (i)

And if we put a-b=0 $\Rightarrow$ a=b, and substitute this to the polynomial, we get:

$f(x) = 0 + (a-c)^3 + (c-a)^3 = (a-c)^3 - (a-c)^3 = 0$

Hence, (a-b) is a factor. $\Rightarrow$ a=b .... (ii)

Substituting (i) in problem polynomial, we get

$f(x) = 0 + (b^3 - 3b^2c + 3\,bc^2 - c^3) + (c^3 - 3c^2a + 3\,ca^2 - a^3)$

$= -3b^2c + 3\,bc^2 - 3ca^2 + 3ca^2$

$= 3(-b^2c + bc^2 - ca^2 + ca^2)$

If we put b-c=0 $\Rightarrow$ b=c, and substitute this to the polynomial, we get:

$f(b=c),\ 3(-c^2xc + cxc^2 - cxc^2 + cxc^2) = 0$

Hence, till now factors are 3x(a-b)x(b-c) .... (iii)

Similarly if we had put c=a, we would have got similar result.

So (c-a) is also a factor.....(iv)

From (ii), (iii), and (iv), we get

3(a-b)(b-c)(c-a) is a complete factorization of the given polynomial.

Answer 16.

If p-q is assumed to be factor, then p=q. Substituting this in problem polynomial, we get:

$f(p=q) = (p-r)^3 + (r-p)^3$

$= (p-r)^3 + (-(p-r))^3$

$= (p-r)^3 - (p-r)^3$

$=0$

Hence, (p-q) is a factor.

Answer 17.

If x-y is assumed to be factor, then x=y. Substituting this in problem polynomial, we get:

$f(x=y)= yz(y^2-z^2) + zy(z^2-y^2) + yy(y^2-y^2)$

$= yz(y^2-z^2) + zy(-(y^2-z^2)) + 0$

$= yz(y^2-z^2) - yz(y^2-z^2) = 0$

Hence, (x-y) is a factor.

Answer 18.

If x-3 is assumed to be factor, then x=3. Substituting this in problem polynomial, we get:

$f(3)= 3x3x3 – 3x3 – 9x3 + 9 = 0$

Hence its proved that x-3 is a factor of the polynomial.

Answer 19.

If $x + 1$ is assumed to be factor, then $x = -1$. Substituting this in problem polynomial, we get:

$f(-1) = (-1)\times(-1)\times(-1) - 6\times(-1)\times(-1) + 5\times(-1) + 12 = 0$

Hence $(x+1)$ is a factor of the polynomial.

Multiplying $(x+1)$ by x^2, we get $x^3 + x^2$, hence we are left with $-7x^2 + 5x + 12$ (and 1st part of factor as x^2).

Multiplying $(x+1)$ by $-7x$, we get $-7x^2 - 7x$, hence we are left with $12x + 12$ (and 2nd part of factor as $-7x$).

Multiplying $(x+1)$ by 12, we get $12x + 12$, hence we are left with 0 (and 3rd part of factor as 12).

Hence complete factor is $(x+1)(x^2-7x+12)$.

Further factorizing $(x^2-7x+12)$, we get:

$x^2 - 3x - 4x + 12 = 0$

⇨ $(x-4)(x-3)=0$

Hence answer is $(x+1)(x-4)(x-3) = 0$

Answer 20.

If $5x - 4$ is assumed to be factor, then $x = \frac{4}{5}$. Substituting this in problem polynomial, we get:

$$f\left(\frac{4}{5}\right) = 5 \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) - 4 \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) - 5 \times \left(\frac{4}{5}\right) + 4$$

$$= \frac{64}{25} - \frac{64}{25} - 4 + 4$$

$$= 0$$

Hence $(5x-4)$ is a factor of the polynomial.

Multiplying $(5x-4)$ by x^2, we get $5x^3 - 4x^2$, hence we are left with $-5x + 4$ (and 1st part of factor as x^2).

Multiplying $(5x-4)$ by -1, we get $-5x + 4$, hence we are left with 0(and 2nd part of factor as $-7x$).

Hence complete factor is $(5x-4)(x^2-1)$.

Further factorizing (x^2-1), we get:

$\Rightarrow (x-1)(x+1)=0$

Hence answer is $(5x-4)(x-1)(x+1) = 0$

Answer 21.

Given $f(x) = (x-1)(x-2)+(-2x+5)$

$=(x^2-3x+2)+(-2x+5)$

$f(x)=x^2-5x+7$

Substituting x=1

$f(x) = 1-5+7 = 3$

when f(x) is divided by (x-1), remainder = 3

substituting x=2

$f(x) = 4-10+7 = 1$

when f(x) is divided by (x-2), remainder = 1

$$\frac{x^2-5x+7}{x^2-3x+2} = 1\frac{(-2x+5)}{(x-1)(x-2)}$$

and

when f(x) is divided by (x-1)(x-2), remainder = (-2x+5).

Answer 22.

/hen x + 1 is a factor, we can substitute x=-1 to evaluate values. ...(i)

When x – 2 is a factor, we can substitute x=2 to evaluate values. ...(ii)

When 2x - 1 is a factor, we can substitute $x=\frac{1}{2}$ to evaluate values. ...(iii)

ubstituting (i), we get

$-1) = ax(-1)^4 + (-1)^3 + b(-1)^2 - 4(-1) + c = 0$

$\Rightarrow a + b + c = -3,$

$\Rightarrow a = -b - c - 3$....(iv)

ubstituting (ii), we get

$\Rightarrow f(2) = ax(2)^4 + (2)^3 + b(2)^2 - 4(2) + c = 0$

$\Rightarrow 16a + 4b + c = 0$ ....(v)

ubstituting (iii), we get

$\Rightarrow f(\frac{1}{2}) = ax(\frac{1}{2})^4 + (\frac{1}{2})^3 + b(\frac{1}{2})^2 - 4(\frac{1}{2}) + c = 0$

$\Rightarrow \frac{a}{16} + \frac{b}{4} + c = 2 - \frac{1}{8}$

$\Rightarrow a + 4b + 16c = 30$….(vi)

utting (iv) in (v) and (vi), we get:

$5a + 4b + c = 0$; $\Rightarrow 16x(-b - c - 3) + 4b + c = 0$

$\Rightarrow -12b - 15c - 48 = 0 \Rightarrow 4b + 5c = -16$

$\Rightarrow b = -4 - \frac{5c}{4}$….(vii)

$+ 4b + 16c = 30 \Rightarrow (-b - c - 3) + 4b + 16c = 30$

$\Rightarrow 3b + 15c = 33$ ….(viii)

Putting (vii) in (viii), we get,

$\Rightarrow 3x(-4 - \frac{5c}{4}) + 15c = 33,$

$\Rightarrow$Solving this, we get

$\Rightarrow c = 4$

Putting this value of c in (viii), we get: